

Discounted Cash Flow: theoretical and practical aspects of its use

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Abstract

Objective: This text presents an in-depth analysis of the theoretical and practical aspects of the discounted cash flow method and its applicability in evaluation processes.

Method: As it is a theoretical article, it draws on extensive literature and knowledge accumulated by the author over the years, much of it in undergraduate and postgraduate disciplines.

Results or Discussion: After a thorough exploration of the two pillars of the method - the discount rate and the cash flow - it becomes evident that the main methods widely adopted in the literature offer consistent results, supported by solid theoretical foundations. I bring a critical discussion about three crucial aspects for the practical application of this method. The first aspect highlights the use of multiples, revealing how this approach can, in certain cases, introduce significant distortions in the evaluation results. Next, I address the issue of the capital cost of free cash flow, demonstrating the importance of calculating this metric on the correct basis, using market values, instead of relying on historical values. Finally, I explore how the method can be effectively used in the value creation process, highlighting how this capacity can be perceived through the method's generic expressions.

Contributions: It is clear that, despite advances in research, there is still ample space in the literature for a more detailed and specific analysis of the challenges and opportunities associated with the use of discounted cash flow. This text reinforces the continued importance of this approach in the field of financial evaluation and highlights the need for constant improvement to guarantee solid and reliable results.

Keywords: Discounted Cash Flow. Discount Rate. Valuation.

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Introduction

Discounted cash flow has been a method extensively used in accounting and financial literature to determine the value of assets and liabilities (Parker, 2013). A query of article repositories, in any language, will result in thousands of results. Even centuries after its first use, it is still possible to discuss this method and make contributions to the academic literature in the area (Kruschwitz & Löffler, 2006). In this sense, it is possible to note the existence of an extensive application of the method (for example Schauten et al., 2010 and Amador, 2021), combined with the observation of errors on the part of users (Fernandez, 2019a).

Specifically in the process of evaluating a company, the use of discounted cash flow can be applied in several ways (Brotherson et al., 2014). This text will present a formalization of the use of discounted cash flow, demonstrating that the main methods existing in the literature are interconnected and can produce the same result, as long as they are properly applied. This formalization is not commonly found in the literature in the area and can be used to understand specific and practical situations for those who wish to use this tool.

The purpose of the article involves not only demonstrating the connection between the main methods, but also pointing out that some of them, used in practical evaluation processes, have theoretical problems and can lead to evaluation errors. In this sense, I discuss the use of multiple and the application of historical weights in determining the weighted cost of capital, called WACC in the literature. I show that the error resulting from using the book value to calculate the discount rate weights can influence the value.

As it is a theoretical article, it draws on extensive literature and knowledge accumulated by the author over the years, much of it in undergraduate and postgraduate subjects. I believe that the content expressed here could be useful for the accounting area for several reasons. Just to cite one example, the recoverability test, present today in Brazilian accounting, may require knowledge that I present here. Accounting is an important source of information for the evaluation process, through financial statements and historical data (for example, Martins, 2000). The specialization of professionals in the area allows them to be in a fundamental position to participate in the process, including due diligence processes (Howson, 2017).

In the next item, I present the formal development of evaluation methods, presenting the fundamental concepts (cash flow and discount rate, as well as value), which will allow the demonstration of existence of similarity in the results of the main evaluation methods.

2 Formal development of methods

2.1 Initial Concepts

The basic expression of discounted cash flow (Dulman, 1989) is as follows:

$$V_0 = \sum_{t=1}^n \frac{FC_t}{(1+i)^t} \quad (1)$$

where V = value at the time of evaluation, assumed here as zero; FC = cash flow, as detailed below; t = period of time; n = time horizon, and I am assuming that in continuity it would be equal to infinity; and i = the discount rate.

The expression is the most basic formula for discounted cash flow and deserves a quick analysis in its two main components: cash flow (FC) and the discount rate (i). Initially, I would like to point out that the value of a company will be directly proportional to the amount of cash flow generated and inversely proportional to the discount rate. In other words, the greater the cash generated and the lower the rate, the greater the value. Another way to apply expression (1) is to consider that the value creation process in a company can occur by increasing the cash flow generated or by reducing the discount rate, or, in cases where the project has a time horizon, by lengthening n.

An additional aspect is that the value expression is generally used with projected values. However, it may be possible to use it for historical data. I will not differentiate this, in this text, but despite the literature associating value with the future, there are several accounting applications where it is possible to apply it to historical data. This can occur, for example, in expert calculations (Mondandon et al., 2008).

2.1.1 Cash Flow

A company is made up of economic resources that will be used to produce wealth. To make this possible, the company will raise funds from its shareholders and expensive capital financiers. In its most basic accounting expression, economic resources are called assets, and sources of financing are called net equity and liabilities. This translates into the basic accounting equation. For the purposes of the evaluation process, the asset is numerically a little different, since non-onerous financing of the liability is considered as a reduction of the asset, as it reduces the need of the company's working capital. Some specific assets can be reclassified as liabilities, as is the case with amounts receivable given as collateral or the credit balance of the bank account. There is literature that briefly addresses this subject (Fiechter, 2011). In this article, I will use the terms in the sense used in finance.

Associated with each component of the accounting equation, there are specific cash flows (Fernandez, 2019b). A company's set of assets will generate a cash flow, which is more commonly called free cash flow. Associated with liabilities, there is the debt cash flow. Finally, cash inflow

and outflow transactions with shareholders are called, not at all creatively, shareholder cash flow. The shareholders' cash flow (FCA) can be obtained through the following expression:

$$FCA_t = LL_t + DD_t + \Delta CG_t + \Delta CT_t + \Delta ANC_t \quad (2)$$

where FCA = shareholders' cash flow, LL = net profit for the year, DD = depreciation and other non-disbursable expenses; ΔCG = variation in working capital; ΔCT = variation in third-party capital; ΔANC = variation in non-current assets; and t = time period t ; and Δ = variation. Free cash flow (FCL) is obtained from the company's operating results. By operating result (LO in the following expression), understand the company's result before remuneration to third-party capital (financial expenses) and after financial income. Basically:

$$FCL_t = LO_t + DD_t + \Delta CG_t + \Delta ANC_t + IR_t \quad (3)$$

And IR represents the income tax for the period. The debt flow (FCD) corresponds to the following expression:

$$FCD_t = \Delta CT_t + DF_t \quad (4)$$

onde ΔCT = variaçãõ do capital de terceiros e DF = despesa financeira.

It is important to note here that the shareholders' cash, as expressed in (2), has already considered, in its net profit, the portion of income tax, as determined by the company's accounting. This value is generally different from the tax (IR) considered in expression (3), since here we have the tax that is levied on profit before financial expenses. I will discuss this tax issue when I comment on adjusted present value.

2.1.2 Discount Rate

The second relevant element in the basic expression of the discounted cash flow formula is the discount rate. This rate reflects the cost of liquidity over time, in addition to the risk for the capital provider (Frederick et al., 2002; Newell & Pizer, 2003; Read, 2001).

In the case of third-party capital, this discount rate generally corresponds to the cost of the resources raised by the company. This value is present in the contracts made between the company and the financier. The determination of this rate must take into account the conditions imposed by whoever provides the resources. As a typical company may have more than one liability, the discount rate must reflect the average of the contracts. In practical terms, each contract may have different indexes, with covenants that need to be taken into consideration (Spiceland et al., 2016).

The discount rate for shareholder resources presents a greater challenge, since there is no explicit contract in

which the return provided to this source of resources is agreed. For practical situations, there are several models that allow estimating the value of this discount rate, such as asset pricing models and other derivatives.

The important thing is that each of the cash flows indicated above needs to be discounted at the appropriate rate. This means that you must use the shareholders' discount rate to discount the FCA and the debt rate for the FCD. In the case of the FCL, the discount rate must be a weighted average of the discount rates, where the weight used corresponds to the market value of the debt and equity. I highlighted the term 'market value', since it has been very common to use other weightings, especially the book value of debt and equity, which is a mistake. I will comment on this in item 3.2, below.

For the purposes of this article, I will call k_d the debt discount rate, k_e the shareholder cash flow discount rate, and k the free cash flow discount rate.

Before proceeding, it is important to highlight that the expression of discounted cash flow (1) is presented through discrete capitalization. In continuous capitalization, the values are calculated at each instant of time, using Euler's constant and, given that the period tends to zero. In capitalization, values are added at specific time intervals, through the expression of compound interest. Despite the elegance of continuous capitalization, its use presupposes perfect liquidity, which takes it away from the real world. Thus, the use of expression (1), with discrete capitalization, comes closer to the reality of the cash flow between the company and its investors, shareholders or not (Ruttiens, 2013).

2.1.3 Value of a Company

The first way to obtain the value of a company is through discounting the free cash flow (FCL) by the weighted average of the discount rates (k). A second way is to use shareholder cash flow (FCA), discounted to the return expected by investors, or k_e . I will develop these two approaches more appropriately in subsection 2.2 below.

There are other methods discussed in the literature, such as aggregate economic value, CFROI (Madden, 1999), adjusted present value, multiple methods, among others. I will address the methods that produce the same results as those mentioned in the previous paragraph in part of item 2 (Fernandez, 2007). The multiples method is widely used in practical evaluations, but it can be a source of inappropriate calculations, which will be discussed in section 3.1.

2.2 Value of a Company by Shareholder Flow and Free Cash Flow

Initially, I will consider a perpetuity situation, with a zero tax

rate, no change in the capital structure and no income tax. In situations of perpetuity (for example Sabal, 2013 and da Cunha, Iara & Rech, 2014), a specific case of financial mathematics, the present value is given by the division of the cash flow at the discount rate. I am assuming a cash flow growth rate of zero. If we are assuming perpetuity, this also means that the company will only invest the amount corresponding to depreciation, which means that $DD_t = \Delta ANC$. Furthermore, this means that there will be no change in the company's working capital, that is, $\Delta CG_t = 0$. The FCA's expression will be as follows:

$$FCA_t = LL_t \quad (2a)$$

Which means:

$$FCA_t = LO_t - k_d D_{t-1} \quad (2b)$$

where k_d corresponds to the debt discount rate and D is the debt.

The value of the company will be determined by discounting the shareholders' cash flow by the cost of equity capital. Since I am dealing with perpetuity, this means that $FC1 = FC2 = \dots = FC\infty$

Thus,

$$V_0 = (FCA/k_e) + D_0 \quad (5)$$

Replacing (2b) in expression (5):

$$V_0 = (LO_t - k_d D_{t-1})/k_e + D_0 \quad (5a)$$

The other way to determine a company's value is through free cash flow. In this case, it is necessary to use a discount rate that reflects the weighted average of a company's two sources of financing: third-party capital (or debt) and equity (for example, Hulten, 1991, and Bruner et al., 1998). I have previously defined the discount rate as:

$$k = (D/V)k_d + (E/V)k_e \quad (6)$$

Where D = debt value and E = market value of equity (Ve'lez-Pareja & Tham, 2009). If the methods are the same, I can make the free cash flow discounted by the rate obtained in (6) equal to (5a):

Once

$$\frac{LO_1 - k_d D_0}{k_e} + D_0 = \frac{LO_1}{k} \quad (7)$$

Replacing (8) in (6) I have:

$$k = k_d \frac{D_0}{\frac{LO_1 - k_d D_0}{k_e} + D_0} + k_e \frac{\frac{LO_1 - k_d D_0}{k_e}}{\frac{LO_1 - k_d D_0}{k_e} + D_0}$$

$$k = \frac{k_d D_0 k_e}{LO_1 - k_d D_0 + k_e D_0} + \frac{(LO_1 - k_d D_0) k_e}{LO_1 - k_d D_0 + D_0 k_e}$$

$$k = \frac{LO_1 k_e}{LO_1 - k_d D_0 + D_0 k_e} \quad (6a)$$

$$E_0 = (LO_1 - k_d D_0)/k_e \quad (8)$$

Substituting the value of (6a) into

$$\frac{LO_1 - k_d D_0}{k_e} + D_0 = \frac{LO_1}{\frac{LO_1 k_e}{LO_1 - k_d D_0 + D_0 k_e}} \quad (7a)$$

$$\frac{LO_1 - k_d D_0}{k_e} + D_0 = \frac{LO_1 - k_d D_0 + D_0 k_e}{k_e} \quad (7b)$$

It is easy to see that both sides of the expression are identical, which proves that determining the value of a company by shareholders' cash flow or free cash flow will lead to the same value, at least in situations of perpetuity.

2.3 Presence of income tax

Now it is possible to relax one of the assumptions in the previous item, which is the non-existence of income tax. The issue of taxes is also present in the company valuation literature (for example, Beatty et al., 1999). Admitting that the tax will be calculated on the company's operating profit, after the financial expense, this means that the expression of the weighted average cost (6) will have the following form:

$$k = (D/V)k_d(1 - a) + (E/V)k_e \quad (6b)$$

where k = weighted cost of capital, D = debt, V = company value, k_d = cost of debt, a = tax rate, E = market value of equity and k_e = cost of own capital. It is possible to see that higher rates tend to reduce the value of k and this influences the value of the company, as shown in expression (1). Substituting (8) into (6b) we have:

$$k = k_d \frac{D_0(1 - a)}{D_0 + \frac{(LO_1 - k_d D_0)(1 - a)}{k_e}} + k_e \frac{(LO_1 - k_d D_0)(1 - a)/k_e}{D_0 + \frac{(LO_1 - k_d D_0)(1 - a)}{k_e}} \quad (6c)$$

$$k = \frac{k_d D_0 k_e (1 - a)}{(LO_1 - k_d D_0)(1 - a) + k_e D_0} + \frac{(LO_1 - k_d D_0)(1 - a) k_e}{(LO_1 - k_d D_0)(1 - a) + k_e D_0}$$

Resulting in

$$k = \frac{LO_1 k_e (1 - a)}{(LO_1 - k_d D_0)(1 - a) + D_0 k_e} \quad (6d)$$

Expression (7) for a non-zero tax rate corresponds to:

$$\frac{(LO_1 - k_d D_0)(1 - a)}{k_e} + D_0 = \frac{LO_1(1 - a)}{k} \quad (7c)$$

Substituting the value of k from expression (6d) into (7c) we have:

$$\frac{(LO_1 - k_d D_0)(1 - a)}{k_e} + D_0 = \frac{LO_1(1 - a)}{\frac{LO_1 k_e (1 - a)}{(LO_1 - k_d D_0)(1 - a) + D_0 k_e}} \quad (7c)$$

Rearranging the second term:

$$\frac{(LO_1 - k_d D_0)(1 - a)}{k_e} + D_0 = \frac{LO_1 - k_d D_0(1 - a)}{k_e} + \frac{D_0 k_e}{k_e}$$

It is easily seen that considering the tax rate does not influence the equality of a company's value according to the two models (shareholders' cash flow and free cash flow).

2.4 Adjusted Present Value

This method was developed by Myers (Myers, 1974) and popularized a few years later in an article published in the Harvard Business Review (Luehrman, 1997). The value of a company is given by the free cash flow without the presence of debt added to the benefit of debt, being discounted at an appropriate rate, k_* . In practical terms, the benefit of debt is given by the fact that a company, when using third-party capital, will have an expense that is deductible from income tax. This benefit, in a perpetuity situation, is represented by aD_0 . Thus, the higher the tax rate in a given country, the greater the debt benefit, which means an incentive for the use of third-party capital. In this way, the value of the company will be given by:

$$V_0 = \frac{LO(1 - a)}{k_*} + aD_0 \quad (9)$$

in accordance with the perpetuity conditions presented above. The value of the discount rate for a company with debt corresponds to:

$$k_* = k \frac{E_0 + D_0}{E_0 + D_0(1 - a)} \quad (10)$$

Necessarily $k > k_*$. In fact, given expression (10) I can say that the higher the rate applied, the higher the value of k_* . Intuitively, I can also say that, in expression (9), the first term will be negatively affected by the increase in the tax rate and the second term positively. It remains to be seen whether we have a compensating mechanism for these effects.

Equation (8), adjusted for income tax, is indicated below:

$$E_0 = (LO_t - k_d D_0)(1 - a)/k_e \quad (8a)$$

Incorporating (6d) into (10):

$$k_* = \frac{LO_1 k_e (1 - a)}{(LO_1 - k_d D_0)(1 - a) + D_0 k_e} \frac{E_0 + D_0}{E_0 + D_0(1 - a)}$$

Now replacing E_0 :

$$k_* = \frac{LO_1 k_e (1 - a)}{(LO_1 - k_d D_0)(1 - a) + D_0 k_e} \frac{(LO_t - k_d D_0)(1 - a)/k_e + D_0}{(LO_t - k_d D_0)(1 - a)/k_e + D_0(1 - a)}$$

Making some adjustments:

$$k_* = \frac{LO_1 k_e (1 - a)}{(LO_1 - k_d D_0)(1 - a) + D_0 k_e} \frac{(LO_t - k_d D_0)(1 - a) + D_0 k_e}{(LO_t - k_d D_0)(1 - a) + D_0(1 - a)k_e}$$

$$k_* = \frac{LO_1 k_e (1 - a)}{(LO_1 - k_d D_0)(1 - a) + D_0 k_e (1 - a)} \quad (10a)$$

Replacing (10a) into (9):

$$V_0 = \frac{LO(1 - a)}{\frac{LO_1 k_e (1 - a)}{(LO_1 - k_d D_0)(1 - a) + D_0 k_e (1 - a)}} + aD_0$$

$$V_0 = \frac{(LO_1 - k_d D_0)(1 - a) + D_0 k_e (1 - a)}{k_e} + aD_0$$

$$V_0 = \frac{(LO_1 - k_d D_0)(1 - a)}{k_e} + D_0(1 - a) + aD_0$$

$$V_0 = \frac{(LO_1 - k_d D_0)(1 - a)}{k_e} + D_0$$

This corresponds to the right-hand side of (7c). This means that the APV equation is coherent with the two models presented previously.

2.5 Economic Profit

An additional way to determine the value of a company involves the use of economic profit and related methods. The economic profit is calculated from the accounting profit, adjusted by the investment remuneration, brought to present value. This present value must be added to the book value of the assets and the value of the debt. There are two main approaches to this analysis. The first consists of using the operating profit and calculating the remuneration of the asset, represented by k . The second approach involves the use of net profit and the evaluation of the remuneration of equity, obtained by multiplying the accounting net equity by k_e .

In the literature, the economic profit method became known as Economic Value Added or EVA (see, for example, Sharma & Kumar, 2010, Shrieves & Wachowicz, 2001,

Biddle et al., 1999, Chen & Dodd, 1997). In algebraic terms:

$$V_0 = \frac{(LO_1 - k_d D_0)(1 - a) - k_e PL_0}{k_e} + PL_0 + D_0 \quad (11)$$

In which PL represents equity, as measured by accounting. The premise underlying this approach is that the company's value will be established based on the shareholders' cash flow, deducting an expected remuneration from them.

The intriguing aspect of this method lies in its use of accounting values during the evaluation process. However, in essence, this method is equivalent to those previously presented:

$$V_0 = \frac{(LO_1 - k_d D_0)(1 - a)}{k_e} - \frac{k_e PL_0}{k_e} + PL_0 + D_0$$

And it is easy to verify the identity claim with the other methods.

2.6 General Case

I used the perpetuity model to perform the previous demonstrations, but it is possible to demonstrate equality between the methods using a specific scenario. The demonstration of the general case is more complex and uses mathematical induction. It assumes that, in situations in which the cash flow varies each period, as does the discount rate, and when the time horizon is not infinite, the value in the base period corresponds to the cash flow of the following period, discounted, added to the value of the immediately subsequent period.

Despite the importance of proving equality between methods in a general scenario, I recognize that there are other issues that are equally or even more relevant that deserve to be discussed in this text. These questions involve the relationship between the theory, as presented previously, and its application in practical situations.

3 Practical assessment situations

In practice, it is common to observe a discrepancy between the application of the discounted cash flow method and the choice of alternative methods by analysts. This difference may arise due to lack of knowledge of the most appropriate technique or the attempt at simplification. In this context, I will highlight how these discrepancies can affect the results, using two scenarios that I consider particularly relevant in the company evaluation process. The first of these concerns the use of multiples, followed by the inadequate weighting of factors, with a special focus on the inclusion of accounting values in equation (6).

3.1 Multiples

The use of multiples has been a common practice in company valuation processes (see, for example, Allee et al., 2020, Damodaran, 2012, Lee, 1999 and Silva & Fernandes, 2016). As the term itself suggests, this approach involves the multiplication of a specific parameter to estimate the value of a company, which may be the number of customers, installed capacity, revenue, among others. The variety of multiples used is considerable, and the practical application may vary according to the sector, the company responsible for the evaluation, the country, and so on. The literature lacks a broad survey of multiples, but it is important to note that widely used indices, such as P/L (Price/Profit), are an example of a multiple restricted to companies with shares traded on an exchange (Koller et al., 2005).

In this context, I propose a discussion that covers two multiples widely adopted in practice: the revenue multiple (Fernandez, 2019b) and EBITDA (Profit before interest, Taxes, Depreciation and Amortization) (Liu et al., 2002). The discussion can be carried out with other multiples, but the choice I make arises from, in addition to the use in situations involving company evaluation, the fact that it is possible to link with expression (1). Through these examples, I intend to highlight the risks associated with the use of this evaluation method, due to its lack of rigor and the need to understand its limitations before applying it. Even with these caveats, multiples continue to be used in practice due to their simplicity and the ability to serve as a parameter for comparison and validation of results obtained by more rigorous methods.

3.1.1 Revenue

Expression (7) can be presented as follows :

$$V_0 = \frac{LO}{K} = \frac{Revenue - expense}{K} \quad (7d)$$

Cost accounting distinguishes between fixed and variable expenses. If we assume that the fixed portion is equal to zero, we can consider that the company's value will be determined by:

$$V_0 = \frac{(1 - \gamma)Revenue}{K} \quad (7e)$$

where γ corresponds to the portion of the company's variable expenses. Considering that the multiple represents a multiplicative factor that will be applied to the revenue to determine the company's value, we can deduce that $(1 - \gamma)/k$, from expression (7e) the multiple that will be applied by the analyst.

This situation illustrates the problem inherent in the use of multiples in evaluation processes. Despite its simplicity

and wide practical application, the final result may be subject to inaccuracies. If fixed expenses are identical for all companies, a compensation process could occur in γ .

The process of constructing a multiple involves using a sample of past transactions as a basis. After obtaining γ , this multiple is applied to new cases. If the cost structure of the company where the multiple is applied presents fixed costs significantly different from that in which the multiple was originally derived, the resulting value V_0 will be subject to bias.

One way to analyze this question is to check the level of error produced by (7e). For this, we define the error as being the difference between the value obtained by the discounted cash flow and the value obtained in (7e). For example, making the difference between (7) and (7e):

$$Error = \frac{Revenue - expense}{K} - \frac{(1 - \gamma)Revenue}{K} \quad (12)$$

which corresponds to $(\gamma Revenue - Expense)/k$.

3.1.2 Ebitda

The use of EBITDA (Earnings before interest, taxes, depreciation and amortization) as a multiple is more subtle than revenue (Brockman and Russell, 2012). Unlike the expression of free cash flow, as presented in (3), EBITDA does not take into account variations in working capital, net investments after depreciation and income tax. Likewise, EBITDA does not correspond to operating profit and cannot be considered as a substitute for operating profit in the previous expressions. In perpetuity situations, such as those used in a simplified manner in this article, (a) investments must correspond to depreciation¹ and (b) variations in working capital will be equal to zero. The absence of income tax can be compensated by the discount rate or, in this case, by the multiple. In this way, the use of a multiple can approximately estimate the value of the company in perpetuity, as long as there is this compensation.

The challenge lies in situations where the cash flow does not resemble a perpetuity, whether due to variations over time, a finite time horizon or the existence of distinct rates discount for each period, as is common in the discounted cash flow method. In these circumstances, when disregarding income tax and investments made - both in non-current assets and in working capital - the use of EBITDA may deviate from the company's real value. The magnitude of the error in the assessment will obviously depend on the amounts involved. It is essential to highlight that EBITDA, although it is a financial metric widely used in evaluating companies, cannot be applied indiscriminately.

¹ It is possible to see that the use of EBITDA corresponds to assuming that depreciation and amortization would be zero, which is totally different from assuming that investments correspond to depreciation.

To use it effectively, it is crucial that the user is fully aware of the challenges and limitations inherent to this approach.

When used in isolation and without considering the specific contexts of each company, it can lead to distorted evaluations.

3.2 Historical Weight

The problem associated with using the accounting structure to weight the values of cost k when discounting the free cash flow lies in the discrepancy between these values and their equivalents in the financial market (Fernandez, 2019a). Expression (6) presents the determination of the discount rate for free cash flow, with the condition that E corresponds to the market value of equity and V to the value of the company. However, in many practical applications, it has become common to use net equity and accounting assets for this purpose.

The bias resulting from this use will depend, of course, on the difference between accounting values and market values, as well as the discrepancies between costs k_d and k_e . So, instead of (6), there is the alternative use of:

$$k = (D/A)k_d + (PL/A)k_e \quad (13)$$

where A corresponds to the value of the asset and $(A = P L + D)$.² Since the error is the difference between using (6) and (13) we have:

$$Error = \frac{LO}{(D/V)k_d + (E/V)k_e} - \frac{LO}{(D/A)k_d + (PL/A)k_e} \quad (14)$$

The level of error obtained in equation (14) will therefore depend on the weights and the difference between the discount rates. As, in general, $PL < E$, other variables remaining unchanged, it follows that the use of accounting values tends to cause the value obtained in (13) to be lower than that in (6). Given that the error is calculated according to expression (14), this means that the use of accounting values tends to underestimate the value of k , which, in turn, will increase the value found by the free cash flow at the same time when using a discount rate (14). This will make the result obtained different from that when using discounted shareholder cash flow.

3.3 Appendix

Here I present an example to make the proof presented in the text more didactic. Consider a company evaluation process, where the following balance sheet projection was

² As I highlighted in item 2.1.1, I am considering here the concept of assets as being deducted from operating liabilities.

made:

Balance	31/12/x0	31/12/x1	31/12/x2
Working Capital	40	30	60
Equipment	80	90	100
Accumulated Depreciation	0	-10	-20
Asset	120	110	140
Debt	70	60	80
PL	50	50	60
Total	120	110	140

Free Cash Flow	x0	x1	x2
Operating Profit	30.00	50.00	60.00
Income Tax	-10.20	-17.00	-20.40
Depreciation Expense	10.00	10.00	10.00
Working Capital Variation	10.00	-30.00	-
ANC Variation	-10.00	-10.00	-10.00
Free Cash Flow	29.80	3.00	36.90

The projected income statement is given by:

Revenue	100	130	150
Operating Expenses	-60	-70	-80
Depreciation Expense	-10	-10	-10
Operating Profit	30	50	60
Financial Expense	-5.60	-5.40	-5.60
Profit Before Tax	24.40	44.60	54.40
Income Taxes	-8.30	-15.16	-18.50
Net Profit	16.10	29.44	35.90

The debt flow (4) is the following:

Debt Flow	x1	x2	x3
Debt Variation	10.00	-20.00	-
Financial Expense	5.60	5.40	5.60
Debt Flow	15.60	-14.60	5.60

Using k_e as a discount rate to discount shareholders' cash flow:

$$V_0 = \frac{16,10}{1,072} + \frac{19,44}{1,072 * 1,0695} + \frac{35,90/0,074}{1,072 * 1,0695} + 70,00 = 525,02$$

The value for other years is relevant for calculating k . Doing then:

$$V_2 = \frac{35,90}{0,074} + 80 = 565,14$$

The parameters used in the table are a tax rate of 34 percent, a k_e and k_d varying over time, as given below:

Parameter	x0	x1	x2
Cost of Debt or k_d	8.00	9.00	7.00
Cost of Equity or k_e	7.20	6.95	7.40

$$V_1 = \frac{(35,90/0,074) + 19,44}{1,0695} + 60 = 531,79$$

From this information it is possible to calculate the shareholders' cash flow, according to expression (2). The values are presented below in tabular form:

These results allow us to calculate the value of k :

FCA	x0	x1	x2
Net Profit	16.10	29.44	35.90
Depreciation Expense	10.00	10.00	10.00
Debt Variation	-10.00	20.00	-
Working Capital Variation	10.00	-30.00	-
ANC Variation	-10.00	-10.00	-10.00
FCA	16.10	19.44	35.90

	x0	x1	x2
Debt	70.00	60.00	80.00
Value	525.02	531.79	565.14
Debt/Value	0.1333	0.1128	0.1415
E/Value	0.8667	0.8872	0.8585
$k_d(1 - a)$	0.0528	0.0594	0.0462
k_e	0.072	0.0695	0.074
k	0.0694	0.0684	0.0701
FCL	29.80	3.00	39.60
V	523.14	531.58	564.91

It is also possible to determine the free cash flow, according to (3):

where $0.0694 = 0.1333*0.0528 + 0.8667*0.072$ and so on.

and $523.14 = (29.80 + 531.58)/1.0731$; $531.58 = (4.84 + 564.91)/1.0718$; and $564.91 = (39.60/0.0701)$. Which show that the method does not change the value obtained in the example

4 Conclusion

Using the tools presented in this article can be very useful for analyzing the source of value creation over time in a company or even in a specific business area (da Cunha, Martins and Neto, 2014). It is good to remember that value is a central issue within finance (Jensen, 2010).

Expression (11) allows us to identify where a company has the ability to add value. This analysis can be valuable in several situations, such as when a company goes through a change in management and the new management seeks to generate greater value for its shareholders. Furthermore, it is useful when carrying out a preliminary analysis before an acquisition process, in order to assess the current value of the target company and its potential to generate future value. These measures drive value and help determine the price to be offered during the purchasing process. From this expression, it is possible to identify several ways to increase value (Koller et al., 2005, Mellen and Evans, 2010, among others):

- Improving the company's operational efficiency, measured by the relationship between revenue generated and operating expenses, as demonstrated in (7d). The focus on increasing revenues results in greater operational profitability and allows the optimization of economies of scale and scope.
- Improving the capital structure, although it is a point of debate in financial theory, especially when considering taxes. The influence of the capital structure on the company's value is a topic under constant discussion.
- Reducing the cost of financing, which can be achieved through several measures, such as improving the disclosure of accounting information, searching for new sources of financing and implementing policy ethics that reduce the risk perceived by creditors (k_d and k_e).
- Reducing investments in capital, which implies producing the same quantity with a leaner volume of assets. This can positively affect asset turnover, reduce the need for working capital in operations and eliminate non-current assets that are not very productive, among other actions.
- Exploring the tax benefits available in legislation, whether by presenting projects to financing agencies with subsidized interest or by implementing ethical and legally sound tax planning.

Thus, value can be increased through various strategies, such as improving operational efficiency, optimizing capital structure, reducing financing costs, improving asset management, taking advantage of tax benefits and adopting best practices that increase profitability. These measures contribute to increasing the company's value. Having a clear understanding of these possibilities is essential for strategic decision-making and maximizing

value for shareholders.

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